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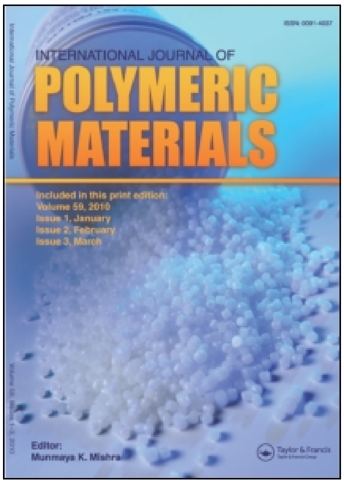

International Journal of Polymeric Materials

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

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To cite this Article Chung, Tai-Shung(1984) 'Effect of Polymer Material Behavior on a Tubular Membrane Deformation', International Journal of Polymeric Materials, 10: 4, 249 — 257

To link to this Article: DOI: 10.1080/00914038408078644

URL: http://dx.doi.org/10.1080/00914038408078644

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Intern. J. Polymeric Mater., 1984, Vol. 10, pp. 249–257
0091-4037/84/1004-0249 $18.50/0

Printed in the United Kingdom

Effect of Polymer Material Behavior on a Tubular Membrane Deformation

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(Received January 11, 1984)

An analysis for describing a tubular membrane expansion under vacuum in a batch air separator is presented. Assuming that the polymer material properties obey an Oldroyd's Fluid B, the effect of rheological properties, relaxation parameters, and surface tension on the membrane inflation are investigated. Numerical results indicate that a viscoelastic membrane with a small Deborah number shortens the transition period of deformation and the magnitude of expansion.

INTRODUCTION

Using extremely thin polymeric films as membranes to separate gas and liquid is a well-established process in the chemical industry. The driving forces for material transportation and separation usually are either pressure or concentration. Since a tubular membrane has the advantage of offering both batch and continuous operation, it is widely used in filtration, dialysis, and reverse osmosis. The membrane life is dependent on the operating and environmental conditions, as well as its own mechanical properties. In most industrial processes, one side of the membrane is operated under vacuum or pressure in order to facilitate the permeation, and this may gradually cause membrane deformation and deterioration due to the external stresses. Therefore, an understanding of the relationship between membrane deformation and its material rheological behavior is very important to prolong its life and reduce the entire operational costs.

Green and Shield[1] were the first to study the deformation of a highly elastic membrane. They considered the uniform expansion of a spherical membrane as an inflation of a thick-walled sphere. Adkins and Rivlin[2] also considered the problem of inflation of a spherical membrane. Foster[3] developed a general solution to describe the very large deformations of axially symmetrical membrane made of neo-Hookean materials. His analysis was based on the assumption that the thickness of the membrane is very small compared to its radius. Later, he and Schaffers[4] extended this theory to study the inflation of symmetrically loaded membranes whose material properties obey the Mooney–Rivlin model. Wineman[5,6] simulated the axisymmetric deformation of non-linear viscoelastic membranes, and proposed various numerical schemes to solve the complicated non-linear equations derived for different material constitutive equations.

The inflation of a tubular membrane can be considered similar to the expansion of a thin cylindrical polymeric film, if the diffusion effect is neglected, and studies of cylindrical viscoelastic film expansion are readily available in literature. Chung and Stevenson[7] investigated the inflation and extension of a tube employing a simplified Lodge rubber-like liquid constitutive equation. Bird *et al.*[8] analyzed a thin spherical film expansion using an Oldroyd Fluid B. Ryan and Dutta[9,10] used modified White–Metzner (MWM) and Zaremba–Fromm–Dewitt (MZFD) models to investigate a parison expansion in the extrusion-blow-molding process.

Most analyses considered the stress built-up within the membranes immediately after the film inflation; however, this may not be the case for some specific chemical processes, such as membranes used in the ionic exchanger, desalination, air separation, and artificial blood vessels. In these cases, the films have been in equilibrium with the surrounding medium for a period of time. This has an important effect on the film expansion if there is an abrupt change in external pressure. In this short paper, we attempt to follow Bird and his coworkers' approach to simulate a cylindrical film deformation using an Oldroyd model B. The effect of material properties, surface tension and the relaxation parameters on the film growth will be addressed.

ANALYSIS

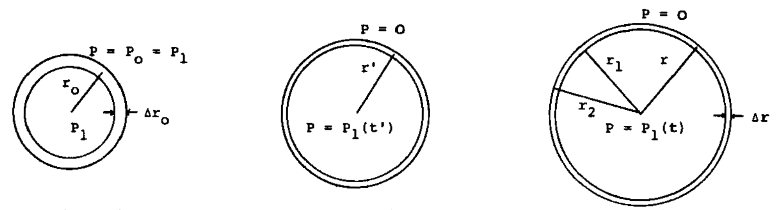
Consider an infinitely long, thin, tubular viscoelastic film used in a batch air separation process, as shown in Figure 1, which has been in equilibrium with the surroundings for a long time. In order to increase separation efficiency, the external medium is suddenly put under vacuum, and the external pressure is removed at time $t=0$. Since the diffusion process is rather slower than the response of a membrane inflation, it is reasonable to assume that the air diffusion process in this transition period is negligible. If the polymeric material is incompressible, the continuity equation and the equation of motion may be expressed as follows:

$$\frac{\partial}{\partial r}(rv)=0 \tag{1}$$

$$\frac{\partial P}{\partial r}=-\frac{\partial}{\partial r}T_{rr}+\frac{T_{rr}-T_{\theta\theta}}{r} \tag{2}$$

Here the film is assumed to grow only radically. Since the driving force is small, the inertial terms are neglected in this analysis. Similar assumptions have been used in the previous reports.[1–4,8] The force balance equations at the internal and external surfaces of the film are:

$$(P+T_{rr})|_{r=r_1}=P_1-\frac{\sigma}{r_1} \tag{3}$$



initial time, t' = o past time, t' > o present time, t

FIGURE 1 Schematic diagram of an inflating film.

and

$$(P+T_{rr})|_{r=r_2} = P_0 + \frac{\sigma}{r_2} \tag{4}$$

Note that the stresses developed in the surrounding phase are neglected in the above equations due to their relatively small magnitude. Substituting Eqs (3) and (4) into Eq. (2) and introducing approximations that the stress tensor is constant across the thin film and $r = r_1$, the yield is:

$$\Delta P - \frac{\sigma}{r}\left(2 - \frac{\Delta r}{r}\right) = (T_{rr} - T_{\theta\theta})\frac{\Delta r}{r} \tag{5}$$

If the viscoelastic behavior of the film obeys the Oldroyd model B in the form,[8]

$$T = -\eta_0 \frac{\lambda_2}{\lambda_1}\dot{\gamma} + \frac{\eta_0}{\lambda_1^2}\left(1 - \frac{\lambda_2}{\lambda_1}\right)\int_{-\infty}^{t} \gamma'_{[0]} \exp[-(t-t')/\lambda_1]\, dt' \tag{6}$$

The normal stress difference between T_{rr} and $T_{\theta\theta}$ may be derived as follows:

$$\begin{aligned} T_{rr} - T_{\theta\theta} &= 4\eta_0\left(\frac{\lambda_2}{\lambda_1}\right)\frac{1}{r}\frac{dr}{dt} + \frac{\eta_0}{\lambda_1^2}\left(1 - \frac{\lambda_2}{\lambda_1}\right) \\ &\quad \times \int_{-\infty}^{t} \left[\left(\frac{r}{r'}\right)^2 - \left(\frac{r'}{r}\right)^2\right] \exp[-(t-t')/\lambda_1]\, dt' \end{aligned} \tag{7}$$

Since the mass balance of polymeric fluid in the film is

$$r_0\, \Delta r_0 = r\, \Delta r \tag{8}$$

and if the equation of state for the air inside the cylindrical membrane is an ideal gas, Eq. (6) may be written as follows with the aid of Eqs (7) and (8):

$$\begin{aligned} \frac{dx}{d\tau} &= \frac{1}{4\beta\left(\frac{\lambda_2}{\lambda_1}\right)x^{\alpha}} - \frac{x^2}{4W_e\beta\left(\frac{\lambda_2}{\lambda_1}\right)}\left(2 - \frac{\beta}{x^2}\right) - \frac{\left(1 - \frac{\lambda_2}{\lambda_1}\right)x}{4\Lambda^2\left(\frac{\lambda_2}{\lambda_1}\right)} \\ &\quad \times \int_{-\infty}^{\tau} \left[\left(\frac{x(\tau)}{x(\tau')}\right)^2 - \left(\frac{x(\tau)}{x(\tau')}\right)^{-2}\right] \times \exp[-(\tau-\tau')/\Lambda]\, d\tau' \end{aligned} \tag{9}$$

Here we have introduced the dimensionless quantities:

$$x=\frac{r}{r_0} \qquad \beta=\frac{\Delta r_0}{r_0} \qquad \alpha=2\gamma-3 \tag{10}$$

$$W_e=\frac{R_0\,\Delta P}{\sigma} \qquad \Lambda=\frac{\lambda_1\,\Delta P}{\eta_0} \qquad \tau=\frac{t\,\Delta P}{\eta_0} \tag{11}$$

where γ is the ratio of C_p to C_v.

RESULTS AND DISCUSSION

The first term in Eqs (9) and (12) represents the driving force for the tube expansion. The second term describes the surface tension acting as a resistance to the film growth. The last term represents the stress developed in the film which provides an additional resistance to expansion. Since Eqs (9) and (10) are non-linear equations, they were solved using a fourth-order Runge–Kutta method. The numerical scheme was tested against the previous work of Bird *et al.* on bubble growth of a viscoelastic film. The agreement between the two is very satisfactory.

Figures 2 and 3 illustrate the reduced radius $x(\tau)$ as a function of the reduced time τ with different λ_2/λ_1 values for both isothermal and adiabatic expansions. Here the polymeric film obeys a codeformational Maxwell model if $\lambda_2/\lambda_1=0$, and it becomes a Newtonian fluid if $\lambda_2/\lambda_1=1$. Both figures demonstrate that a viscoelastic film expands faster than a purely viscous film in the beginning of inflation. This is in agreement with Ryan and Dutta's theoretical results where they studied the growth behavior of a thin polymeric parison in the extrusion blow-molding process using MZFD and MWD fluid models.[9,10] These figures also show that the viscoelastic film expands slower than a Newtonian film of the same zero-shear-rate viscosity at later times. A similar observation has been reported by Bird and co-workers[8] for the inflation of a spherical viscoelastic film described by the same Oldroyd fluid B model. These results clearly indicate that a viscoelastic-type membrane has better resistance from pressure variation than a viscous-type.

The effect of the surface tension on the film growth is shown in Figure 2. An increase in the surface tension or a decrease in the

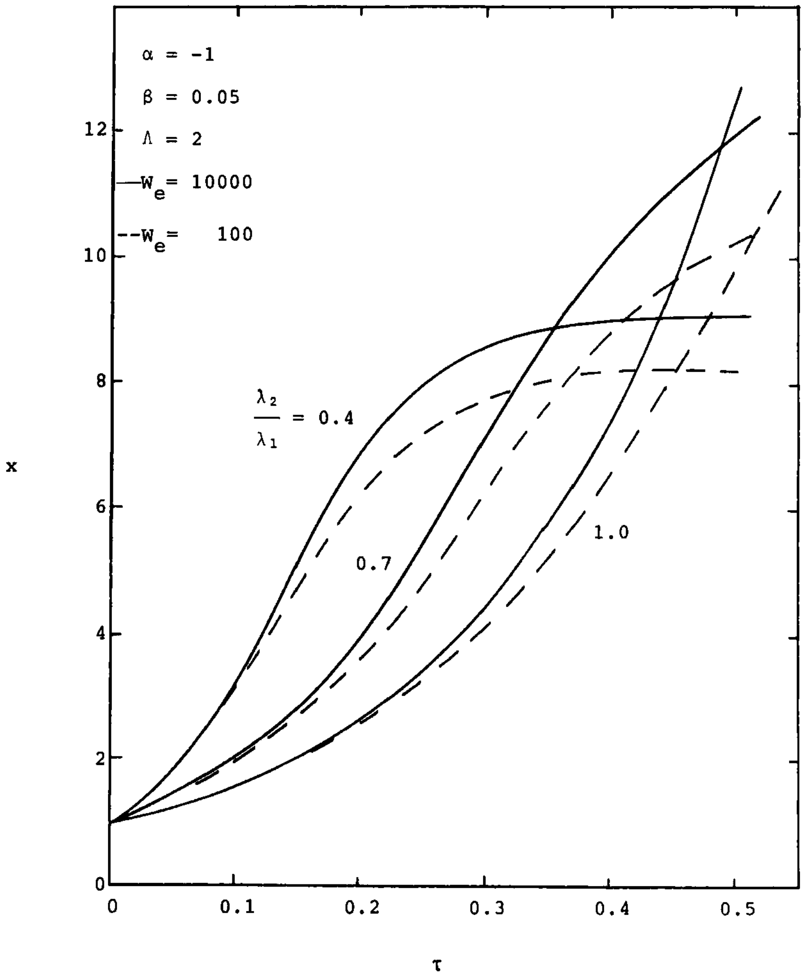


FIGURE 2 Effect of W_e on the isothermal inflation of a cylindrical film.

Weber number, W_e, results in a higher resistance for film inflation. As a result, a liquid-type membrane may have higher resistance from pressure fluctuation if it has better adhesion between membranes and surrounding fluids. However, for most polymeric membranes, the Weber number is around 10^4–10^6; the influence of the Weber number may be neglected. The definition of Λ is somewhat

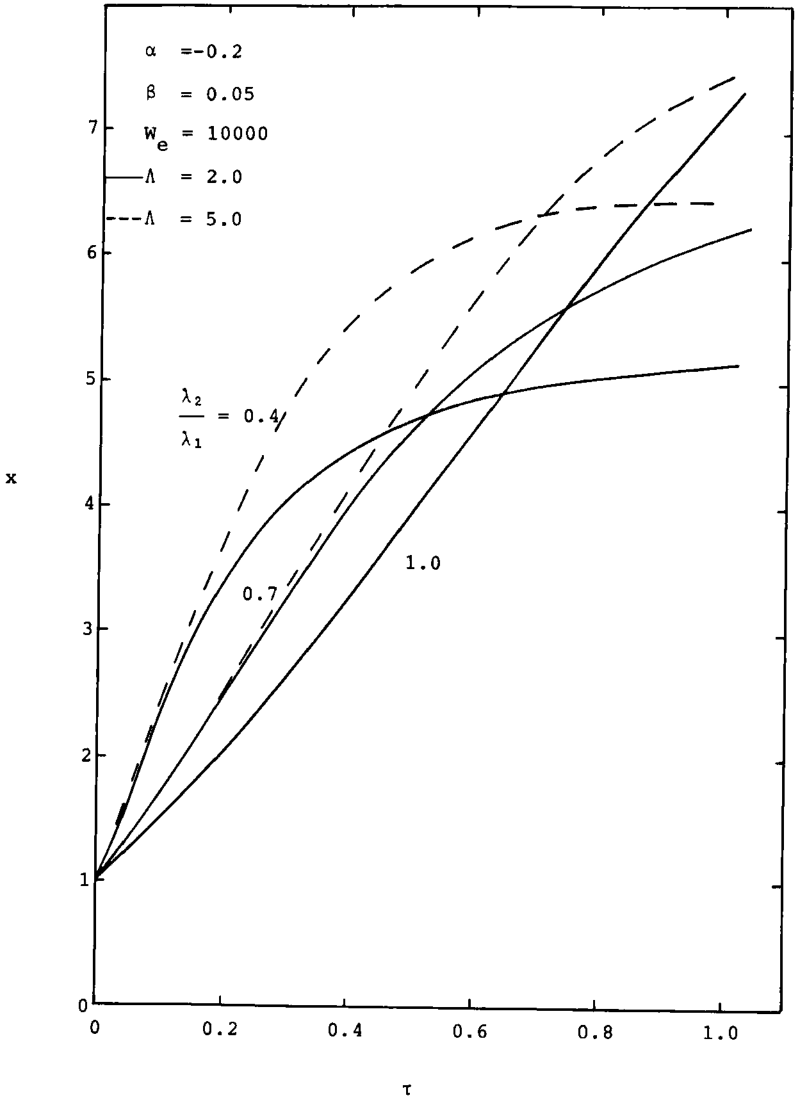


FIGURE 3 Effect of Λ on the adiabatic expansion of a cylindrical film.

similar to the Deborah number. Figure 3 illustrates its effect on film expansion. Since the Deborah number may be defined as a ratio of the relaxation time of the fluid to some appropriate time scale ($\eta_0/\Delta P$) of the deformation,[11] this figure suggests that an increase in Λ prolongs the relaxation phenomena during the inflation process. Therefore, a membrane which has a small Deborah number shortens the transition period of deformation, as well as the magnitude of expansions for a reasonable period of time. From a process standpoint, this type of membrane may be more suitable than others for most separation processes.

NOTATIONS

P = Pressure
P_0 = External pressure
P_1 = Internal pressure
r = Radius
r_0 = Initial radius
r_1 = Internal radius
r_2 = External radius
T = Stress tensor
t = Time
v = Radial velocity
ΔP = Pressure difference across the film, $P_1 - P_0$
Δr = Film thickness
Δr_0 = Initial film thickness
γ = Ratio of heat capacities, C_p/C_v
$\dot{\gamma}$ = Rate-of-strain tensor
$\gamma_{[0]}$ = Strain tensor
η_0 = Zero-shear-rate viscosity
λ_1/λ_2 = Time constants

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